<Plus - Minus>

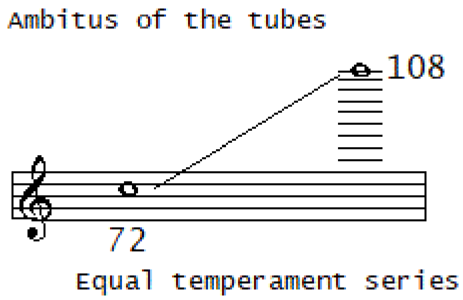
2019

'Plus-Minus', an interactive acoustic audio art project

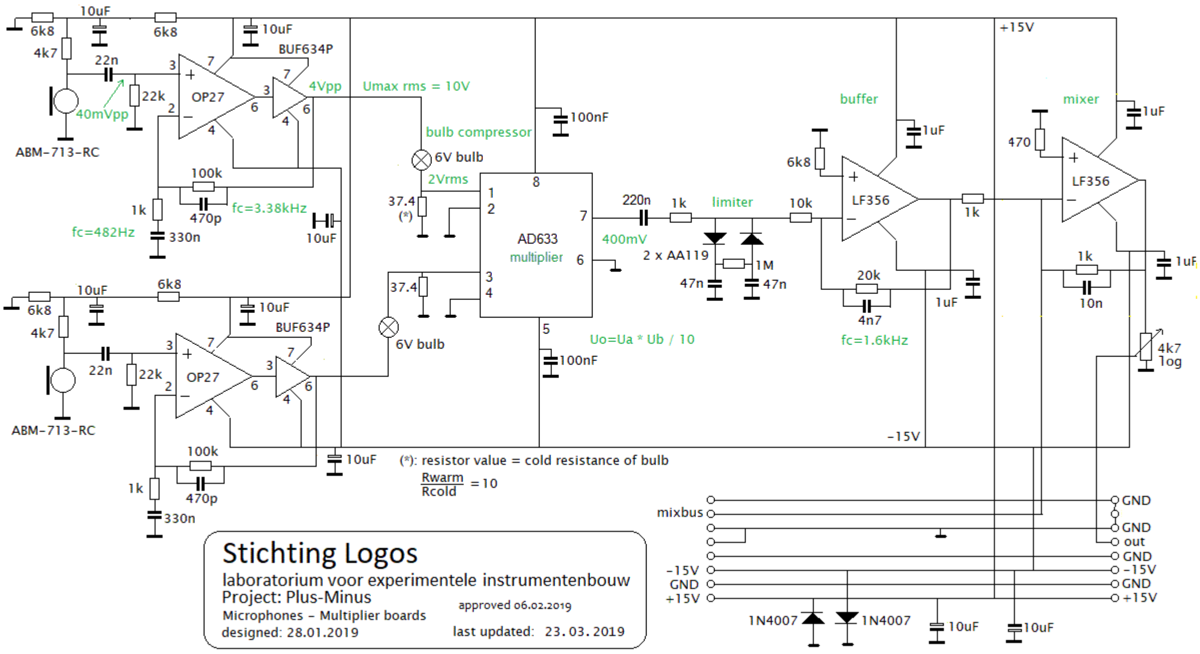
After having build some 76 musical robots, all acoustic in their sound generation but extensively controlled through both digital and analog electronics, the latest project is very different in nature. The robots, no matter how successful they are musically, if used in a non-conventional interactive way -for instance, gesture controlled- with arbitrary audiences, the results were rarely acceptable from a musical point of view. Provided with traditional interfaces (such as keyboards) at the other end, we felt ourselves merely reinventing the piano or for that matter any other existing instrument, thus making the automation of instruments a completely unnecessary mediated step.

'Plus-Minus', in contrast, is an audio art installation allowing participants to explore the world of sum- and difference tones, the base of musical harmony, in a truly interactive way. The musical aesthetics are confined to a very well defined range of possibilities. However, some intelligence is still required to make the artistic result into a seducing and surprising experience. 'Plus-Minus' is an audio art installation allowing participants to explore the world of sum- and difference tones in a truly interactive way. The project consists of 18 analog ring modulators (multiplier circuits) combined with 38 compressors. Purely analog as well. The output is through three large megaphone speakers. The input -and that's were the project really gets to become interactive- is a very large set of precisely tuned thick aluminum tubes, to be freely selected in pairs by the participants.

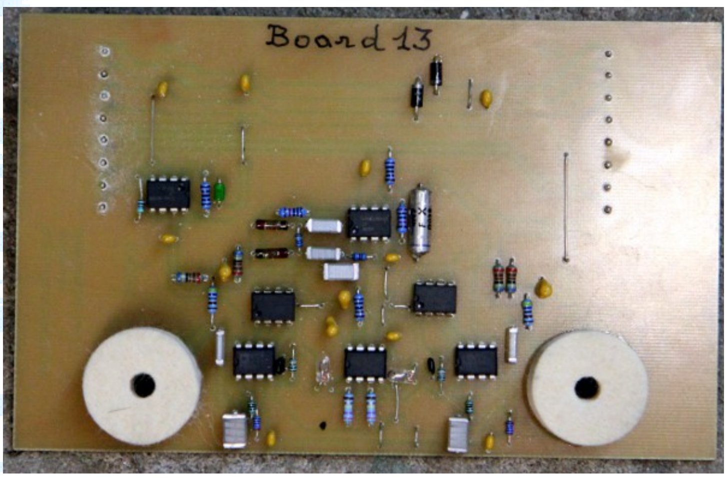
The musical range of the tubes is:



The tubes can be suspended in three rows on a triangular welded structure in stainless steel mounted on three poles. The power amplifiers found a place inside the bases of the poles. Each horizontal carrier accomodates 12 tubes, arranged in pairs. For each pair of tubes, there is an electronic circuit board holding two tiny microphones in immediate proximity to the tubes. The microphone signal is preamplified, filtered and compressed (using a 1:10 bulb compressor) before it is presented to a multiplier, causing the sum and difference tones that will be sounded on striking the tubes. Here is the complete circuit diagram:



A second compressor, using legacy AA119 Germanium diodes, follows the multiplier circuit, thus avoiding the exponential amplitude curve (and the harsh attacks...) that would otherwise be characteristic for multiplier outputs. The single sided and hand made boards -the project has 18 of them- look like this:



The microphones are protected against impacts with 10 mm thick felt rings glued on the PCB. The 18 boards are mounted on the structure with shock absorbing rubber mounts. Thus not only feedback is avoided, but also the ugly sound caused by accidentally hitting the structure itself.

The transfer function of the multiplier, in the time-domain is $U_o = \frac{U_x \cdot U_y}{k}$ The scaling factor k, for the AD633 chips used here is k = 10. The absolute value of Ux and Uy must be <= 10 V. If we feed the circuit with two periodic functions, for instance and to keep math simple, two sinewaves:

$$U_x = U_{xp} \cdot \cos((\omega_x + \phi_x) \cdot t)$$
$$= U_{xp} \cdot \cos((2\pi f_x + \phi_x) \cdot t)$$
$$U_y = U_{yp} \cdot \cos((2\pi f_y + \phi_y) \cdot t)$$

fx and fy are the frequencies and phase angles of the input and Uxp and Uyp the peak amplitudes of both signals. If we now develop the product we get:

$$\text{stel: } \Phi = \phi_x - \phi_y$$

$$U_o = U_{xp} \cdot U_{yp} \cdot (\cos((\omega_x - \omega_y) \cdot t - \Phi) + \cos((\omega_x + \omega_y) \cdot t + \Phi)) \cdot \cos((\omega_x + \Phi) \cdot t)$$
$$= \frac{U_{xp} \cdot U_{yp}}{2} (\cos(\omega_y \cdot t + \Phi) + \cos(-\omega_y \cdot t - \Phi) + \cos((2\omega_x +/- \omega_y) \cdot t +/- \Phi))$$

thus leading to the conclusion that the output signal contains the sum and difference frequencies of the input frequencies. Generalizing for arbitrary periodic waveforms, the result will contain all sum and difference frequencies that can exist in the spectrum between both input signals.

For the sets of tubes we tuned to equal temperament and provide with the project, the sum- and difference tones, expressed as fractional midi-notes, can be found in this table:

	72	73	74	75	76	77	78	79	80	81	82	83	84	85	86	87	88	89
72	0.	23.07	35.62	43.18	48.67	53.04	56.68	59.89	62.77	65.38	67.74	69.92	71.99	73.93	75.78	77.55	79.24	80.86
73	84.5	0.	24.16	36.67	44.19	49.66	54.	57.69	60.93	63.81	66.37	68.72	70.93	72.99	74.94	76.79	78.55	80.24
74	85.02	85.5	0.	25.18	37.66	45.16	50.6	55.	58.72	61.96	64.79	67.35	69.73	71.93	73.99	75.94	77.79	79.54
75	85.56	86.02	86.5	0.	26.14	38.6	46.07	51.59	56.03	59.76	62.94	65.76	68.36	70.72	72.93	74.99	76.94	78.78
76	86.11	86.56	87.02	87.5	0.	27.05	39.49	47.07	52.64	57.08	60.74	63.91	66.78	69.35	71.72	73.93	75.99	77.93
77	86.67	87.1	87.55	88.02	88.5	0.	27.91	40.54	48.16	53.72	58.07	61.72	64.94	67.78	70.36	72.74	74.94	76.99
78	87.24	87.66	88.1	88.55	89.01	89.49	0.	29.14	41.72	49.3	54.73	59.07	62.77	65.95	68.8	71.39	73.75	75.94
79	87.84	88.24	88.66	89.1	89.55	90.01	90.48	0.	30.28	42.82	50.25	55.69	60.09	63.76	66.96	69.81	72.38	74.74
80	88.45	88.84	89.25	89.67	90.11	90.55	91.01	91.49	0.	31.35	43.69	51.16	56.68	61.05	64.74	67.95	70.79	73.36
81	89.08	89.45	89.85	90.26	90.68	91.11	91.55	92.02	92.5	0.	32.03	44.52	52.12	57.62	62.02	65.72	68.92	71.76
82	89.71	90.07	90.45	90.84	91.25	91.67	92.1	92.55	93.02	93.5	0.	33.	45.62	53.14	58.65	63.05	66.73	69.92
83	90.35	90.7	91.07	91.45	91.84	92.25	92.66	93.1	93.55	94.02	94.5	0.	34.22	46.65	54.19	59.69	64.07	67.74
84	91.01	91.35	91.7	92.07	92.45	92.84	93.24	93.67	94.1	94.56	95.02	95.5	0.	35.07	47.62	55.18	60.67	65.04
85	91.68	92.01	92.35	92.7	93.07	93.45	93.84	94.24	94.67	95.11	95.56	96.02	96.5	0.	36.16	48.67	56.19	61.66
86	92.37	92.68	93.01	93.35	93.7	94.07	94.44	94.84	95.25	95.67	96.11	96.55	97.02	97.5	0.	37.18	49.66	57.16
87	93.07	93.37	93.68	94.01	94.35	94.7	95.07	95.45	95.84	96.25	96.67	97.11	97.56	98.02	98.5	0.	38.14	50.6
88	93.78	94.07	94.37	94.69	95.01	95.35	95.7	96.07	96.45	96.85	97.25	97.67	98.11	98.56	99.02	99.5	0.	39.05

	90	91	92	93	94	95	96	97	98	99	100	101	102	103	104	105	106	107	108
72	82.43	83.96	85.45	86.9	88.29	89.67	91.02	92.34	93.64	94.92	96.17	97.41	98.63	99.84	101.03	102.22	103.38	104.54	105.69
73	81.86	83.44	84.97	86.46	87.89	89.3	90.68	92.02	93.34	94.64	95.92	97.17	98.41	99.63	100.84	102.04	103.21	104.38	105.54
74	81.23	82.86	84.45	85.98	87.45	88.89	90.3	91.67	93.02	94.34	95.64	96.91	98.17	99.41	100.63	101.84	103.03	104.22	105.38
75	80.54	82.23	83.88	85.45	86.97	88.45	89.89	91.3	92.67	94.02	95.34	96.63	97.91	99.17	100.41	101.63	102.84	104.03	105.22
76	79.78	81.54	83.25	84.88	86.44	87.97	89.45	90.89	92.3	93.68	95.02	96.34	97.63	98.91	100.17	101.41	102.63	103.84	105.03
77	78.93	80.78	82.56	84.25	85.87	87.45	88.98	90.45	91.89	93.3	94.68	96.02	97.34	98.63	99.91	101.17	102.41	103.63	104.84
78	78.	79.95	81.81	83.57	85.25	86.88	88.46	89.98	91.46	92.9	94.31	95.67	97.02	98.34	99.64	100.92	102.17	103.41	104.64
79	76.94	79.	80.96	82.81	84.56	86.25	87.89	89.45	90.98	92.46	93.9	95.3	96.67	98.02	99.34	100.64	101.92	103.18	104.41
80	75.73	77.93	80.01	81.96	83.79	85.56	87.26	88.88	90.45	91.98	93.46	94.89	96.3	97.67	99.02	100.34	101.64	102.92	104.17
81	74.34	76.71	78.93	81.	82.93	84.78	86.56	88.24	89.87	91.45	92.97	94.44	95.88	97.29	98.67	100.02	101.34	102.64	103.91
82	72.77	75.35	77.74	79.95	81.99	83.94	85.8	87.55	89.25	90.88	92.45	93.97	95.45	96.89	98.3	99.68	101.02	102.34	103.64
83	70.93	73.78	76.38	78.76	80.94	83.	84.96	86.8	88.56	90.26	91.88	93.45	94.97	96.45	97.89	99.3	100.68	102.02	103.34
84	68.72	71.93	74.8	77.38	79.74	81.94	84.01	85.95	87.8	89.56	91.25	92.87	94.44	95.97	97.45	98.9	100.3	101.68	103.02
85	66.05	69.73	72.96	75.81	78.37	80.74	82.95	85.	86.95	88.8	90.56	92.25	93.87	95.45	96.97	98.46	99.89	101.3	102.68
86	62.66	67.04	70.76	73.96	76.79	79.37	81.75	83.94	86.	87.95	89.8	91.55	93.24	94.87	96.45	97.98	99.45	100.9	102.3
87	58.14	63.65	68.07	71.76	74.94	77.79	80.38	82.74	84.94	87.01	88.95	90.79	92.55	94.24	95.88	97.45	98.97	100.45	101.89
88	51.59	59.14	64.69	69.08	72.74	75.94	78.8	81.37	83.74	85.95	88.01	89.94	91.79	93.55	95.25	96.88	98.45	99.97	101.45

89	94.5	94.77	95.07	95.37	95.68	96.01	96.34	96.7	97.07	97.45	97.84	98.25	98.67	99.1	99.55	100.02	100.5	0.	40.12	52.64	60.22	65.72	70.07	73.75	76.96	79.8	82.38	84.75	86.95	89.	90.94	92.79	94.56	96.25	97.88	99.45	100.98
90	95.23	95.5	95.78	96.07	96.37	96.68	97.	97.34	97.7	98.07	98.45	98.84	99.25	99.67	100.1	100.56	101.02	101.5	0.	41.14	53.72	61.24	66.69	71.07	74.77	77.95	80.8	83.39	85.75	87.94	90.	91.95	93.8	95.56	97.25	98.88	100.45
91	95.98	96.23	96.5	96.78	97.07	97.37	97.68	98.	98.35	98.71	99.07	99.45	99.84	100.25	100.67	101.11	101.56	102.02	102.5	0.	42.28	54.73	62.2	67.69	72.09	75.76	78.96	81.81	84.38	86.74	88.94	91.	92.95	94.8	96.56	98.25	99.88
92	96.74	96.98	97.24	97.5	97.78	98.07	98.37	98.68	99.01	99.36	99.71	100.07	100.45	100.85	101.25	101.68	102.11	102.56	103.02	103.5	0.	43.18	55.61	63.16	68.68	73.05	76.74	79.95	82.79	85.36	87.73	89.93	92.	93.95	95.79	97.56	99.25
93	97.5	97.74	97.98	98.24	98.51	98.78	99.07	99.37	99.69	100.02	100.36	100.71	101.08	101.45	101.85	102.26	102.68	103.11	103.56	104.02	104.51	0.	44.03	56.6	64.18	69.66	74.05	77.74	80.94	83.78	86.36	88.73	90.93	93.	94.94	96.79	98.56
94	98.28	98.5	98.73	98.98	99.23	99.5	99.78	100.07	100.37	100.69	101.01	101.35	101.71	102.07	102.45	102.84	103.25	103.67	104.11	104.56	105.02	105.5	0.	45.16	57.69	65.19	70.69	75.08	78.76	81.94	84.78	87.36	89.74	91.95	94.	95.95	97.8
95	99.06	99.28	99.5	99.74	99.98	100.23	100.5	100.78	101.07	101.38	101.69	102.01	102.35	102.71	103.07	103.45	103.85	104.25	104.67	105.11	105.56	106.03	106.5	0.	46.22	58.65	66.19	71.69	76.07	79.74	82.93	85.78	88.37	90.74	92.94	95.	96.95
96	99.86	100.07	100.28	100.5	100.74	100.98	101.23	101.5	101.78	102.08	102.38	102.69	103.02	103.36	103.71	104.08	104.46	104.85	105.25	105.68	106.11	106.56	107.03	107.5	0.	47.07	59.62	67.18	72.67	77.04	80.72	83.93	86.78	89.37	91.74	93.94	96.
97	100.66	100.86	101.06	101.28	101.5	101.74	101.98	102.23	102.5	102.78	103.07	103.37	103.69	104.01	104.35	104.71	105.07	105.45	105.84	106.25	106.68	107.11	107.56	108.02	108.51	0.	48.16	60.67	68.19	73.66	78.05	81.73	84.94	87.8	90.37	92.74	94.94
98	101.48	101.66	101.86	102.06	102.28	102.5	102.73	102.98	103.24	103.51	103.78	104.07	104.37	104.69	105.01	105.36	105.71	106.07	106.45	106.85	107.26	107.68	108.11	108.56	109.03	109.5	0.	49.18	61.66	69.16	74.66	79.04	82.74	85.95	88.79	91.37	93.74
99	102.3	102.48	102.67	102.86	103.07	103.28	103.5	103.74	103.98	104.24	104.51	104.78	105.07	105.37	105.69	106.02	106.36	106.71	107.07	107.45	107.85	108.26	108.68	109.11	109.56	110.03	110.51	0.	50.14	62.6	70.14	75.65	80.05	83.74	86.94	89.79	92.37
100	103.13	103.3	103.48	103.67	103.86	104.07	104.28	104.5	104.74	104.99	105.24	105.5	105.78	106.07	106.38	106.69	107.02	107.36	107.71	108.07	108.46	108.85	109.26	109.68	110.12	110.56	111.03	111.51	0.	51.05	63.59	71.14	76.66	81.06	84.74	87.94	90.79
101	103.97	104.13	104.3	104.48	104.66	104.86	105.06	105.27	105.5	105.74	105.98	106.23	106.5	106.78	107.07	107.37	107.69	108.01	108.35	108.7	109.07	109.45	109.85	110.25	110.68	111.11	111.56	112.03	112.5	0.	52.12	64.64	72.19	77.69	82.07	85.75	88.95
102	104.81	104.97	105.13	105.3	105.48	105.66	105.85	106.06	106.27	106.5	106.73	106.98	107.23	107.5	107.78	108.07	108.37	108.69	109.01	109.35	109.71	110.07	110.45	110.85	111.26	111.68	112.11	112.56	113.03	113.5	0.	53.14	65.67	73.21	78.69	83.07	86.75
103	105.67	105.81	105.97	106.13	106.3	106.47	106.66	106.85	107.06	107.28	107.5	107.73	107.98	108.23	108.5	108.78	109.07	109.37	109.69	110.01	110.36	110.71	111.07	111.45	111.85	112.25	112.68	113.11	113.56	114.02	114.5	0.	54.19	66.69	74.2	79.69	84.07
104	106.53	106.67	106.81	106.97	107.13	107.3	107.48	107.66	107.86	108.07	108.28	108.5	108.74	108.98	109.24	109.5	109.78	110.07	110.37	110.69	111.02	111.36	111.71	112.08	112.46	112.85	113.26	113.68	114.11	114.56	115.02	115.5	0.	55.18	67.65	75.19	80.68
105	107.4	107.53	107.67	107.82	107.97	108.13	108.3	108.48	108.67	108.86	109.07	109.28	109.5	109.74	109.98	110.24	110.51	110.78	111.07	111.37	111.69	112.02	112.36	112.71	113.08	113.46	113.85	114.26	114.68	115.11	115.56	116.03	116.51	0.	56.11	68.64	76.18
106	108.27	108.4	108.53	108.67	108.82	108.97	109.13	109.3	109.48	109.67	109.86	110.06	110.28	110.5	110.74	110.98	111.24	111.5	111.78	112.07	112.38	112.69	113.02	113.36	113.71	114.08	114.46	114.85	115.26	115.68	116.11	116.56	117.03	117.51	0.	57.16	69.66
107	109.15	109.27	109.4	109.53	109.67	109.82	109.97	110.13	110.3	110.48	110.67	110.86	111.07	111.28	111.5	111.74	111.98	112.24	112.5	112.78	113.08	113.38	113.69	114.02	114.36	114.71	115.08	115.46	115.85	116.26	116.68	117.11	117.56	118.03	118.51	0.	58.14
108	110.04	110.15	110.27	110.4	110.53	110.67	110.81	110.97	111.13	111.3	111.48	111.67	111.86	112.07	112.28	112.5	112.74	112.98	113.24	113.5	113.79	114.08	114.38	114.69	115.02	115.36	115.71	116.08	116.46	116.85	117.26	117.68	118.11	118.56	119.03	119.51	0.

The same table, but here expressed as frequencies:

0	523	554	587	622	659	698	739	783	830	880	932	987	1046	1108	1174	1244	1318	1396	1479	1567	1661	1760	1864	1975	2093	2217	2349	2489	2637	2793	2959	3135	3322	3520	3729	3951	4186
523	0	31	64	99	136	175	216	260	307	357	409	464	523	585	651	721	795	873	956	1044	1138	1237	1341	1452	1570	1694	1826	1966	2114	2270	2436	2612	2799	2997	3206	3428	3663
554	1077	0	33	68	105	144	185	229	276	326	378	433	492	554	620	690	764	842	925	1013	1107	1206	1310	1421	1539	1663	1795	1935	2083	2239	2405	2581	2768	2966	3175	3397	3632
587	1110	1141	0	35	72	111	152	196	243	293	345	400	459	521	587	657	731	809	892	980	1074	1173	1277	1388	1506	1630	1762	1902	2050	2206	2372	2548	2735	2933	3142	3364	3599
622	1145	1176	1209	0	37	76	117	161	208	258	310	365	424	486	552	622	696	774	857	945	1039	1138	1242	1353	1471	1595	1727	1867	2015	2171	2337	2513	2700	2898	3107	3329	3564
659	1182	1213	1246	1281	0	39	80	124	171	221	273	328	387	449	515	585	659	737	820	908	1002	1101	1205	1316	1434	1558	1690	1830	1978	2134	2300	2476	2663	2861	3070	3292	3527
698	1221	1252	1285	1320	1357	0	41	85	132	182	234	289	348	410	476	546	620	698	781	869	963	1062	1166	1277	1395	1519	1651	1791	1939	2095	2261	2437	2624	2822	3031	3253	3488
739	1262	1293	1326	1361	1398	1437	0	44	91	141	193	248	307	369	435	505	579	657	740	828	922	1021	1125	1236	1354	1478	1610	1750	1898	2054	2220	2396	2583	2781	2990	3212	3447
783	1306	1337	1370	1405	1442	1481	1522	0	47	97	149	204	263	325	391	461	535	613	696	784	878	977	1081	1192	1310	1434	1566	1706	1854	2010	2176	2352	2539	2737	2946	3168	3403
830	1353	1384	1417	1452	1489	1528	1569	1613	0	50	102	157	216	278	344	414	488	566	649	737	831	930	1034	1145	1263	1387	1519	1659	1807	1963	2129	2305	2492	2690	2899	3121	3356
880	1403	1434	1467	1502	1539	1578	1619	1663	1710	0	52	107	166	228	294	364	438	516	599	687	781	880	984	1095	1213	1337	1469	1609	1757	1913	2079	2255	2442	2640	2849	3071	3306
932	1455	1486	1519	1554	1591	1630	1671	1715	1762	1812	0	55	114	176	242	312	386	464	547	635	729	828	932	1043	1161	1285	1417	1557	1705	1861	2027	2203	2390	2588	2797	3019	3254
987	1510	1541	1574	1609	1646	1685	1726	1770	1817	1867	1919	0	59	121	187	257	331	409	492	580	674	773	877	988	1106	1230	1362	1502	1650	1806	1972	2148	2335	2533	2742	2964	3199
1046	1569	1600	1633	1668	1705	1744	1785	1829	1876	1926	1978	2033	0	62	128	198	272	350	433	521	615	714	818	929	1047	1171	1303	1443	1591	1747	1913	2089	2276	2474	2683	2905	3140
1108	1631	1662	1695	1730	1767	1806	1847	1891	1938	1988	2040	2095	2154	0	66	136	210	288	371	459	553	652	756	867	985	1109	1241	1381	1529	1685	1851	2027	2214	2412	2621	2843	3078
1174	1697	1728	1761	1796	1833	1872	1913	1957	2004	2054	2106	2161	2220	2282	0	70	144	222	305	393	487	586	690	801	919	1043	1175	1315	1463	1619	1785	1961	2148	2346	2555	2777	3012
1244	1767	1798	1831	1866	1903	1942	1983	2027	2074	2124	2176	2231	2290	2352	2418	0	74	152	235	323	417	516	620	731	849	973	1105	1245	1393	1549	1715	1891	2078	2276	2485	2707	2942
1318	1841	1872	1905	1940	1977	2016	2057	2101	2148	2198	2250	2305	2364	2426	2492	2562	0	78	161	249	343	442	546	657	775	899	1031	1171	1319	1475	1641	1817	2004	2202	2411	2633	2868
1396	1919	1950	1983	2018	2055	2094	2135	2179	2226	2276	2328	2383	2442	2504	2570	2640	2714	0	83	171	265	364	468	579	697	821	953	1093	1241	1397	1563	1739	1926	2124	2333	2555	2790
1479	2002	2033	2066	2101	2138	2177	2218	2262	2309	2359	2411	2466	2525	2587	2653	2723	2797	2875	0	88	182	281	385	496	614	738	870	1010	1158	1314	1480	1656	1843	2041	2250	2472	2707
1567	2090	2121	2154	2189	2226	2265	2306	2350	2397	2447	2499	2554	2613	2675	2741	2811	2885	2963	3046	0	94	193	297	408	526	650	782	922	1070	1226	1392	1568	1755	1953	2162	2384	2619

1661	2184	2215	2248	2283	2320	2359	2400	2444	2491	2541	2593	2648	2707	2769	2835	2905	2979	3057	3140	3228	0	99	203	314	432	556	688	828	976	1132	1298	1474	1661	1859	2068	2290	2525
1760	2283	2314	2347	2382	2419	2458	2499	2543	2590	2640	2692	2747	2806	2868	2934	3004	3078	3156	3239	3327	3421	0	104	215	333	457	589	729	877	1033	1199	1375	1562	1760	1969	2191	2426
1864	2387	2418	2451	2486	2523	2562	2603	2647	2694	2744	2796	2851	2910	2972	3038	3108	3182	3260	3343	3431	3525	3624	0	111	229	353	485	625	773	929	1095	1271	1458	1656	1865	2087	2322
1975	2498	2529	2562	2597	2634	2673	2714	2758	2805	2855	2907	2962	3021	3083	3149	3219	3293	3371	3454	3542	3636	3735	3839	0	118	242	374	514	662	818	984	1160	1347	1545	1754	1976	2211
2093	2616	2647	2680	2715	2752	2791	2832	2876	2923	2973	3025	3080	3139	3201	3267	3337	3411	3489	3572	3660	3754	3853	3957	4068	0	124	256	396	544	700	866	1042	1229	1427	1636	1858	2093
2217	2740	2771	2804	2839	2876	2915	2956	3000	3047	3097	3149	3204	3263	3325	3391	3461	3535	3613	3696	3784	3878	3977	4081	4192	4310	0	132	272	420	576	742	918	1105	1303	1512	1734	1969
2349	2872	2903	2936	2971	3008	3047	3088	3132	3179	3229	3281	3336	3395	3457	3523	3593	3667	3745	3828	3916	4010	4109	4213	4324	4442	4566	0	140	288	444	610	786	973	1171	1380	1602	1837
2489	3012	3043	3076	3111	3148	3187	3228	3272	3319	3369	3421	3476	3535	3597	3663	3733	3807	3885	3968	4056	4150	4249	4353	4464	4582	4706	4838	0	148	304	470	646	833	1031	1240	1462	1697
2637	3160	3191	3224	3259	3296	3335	3376	3420	3467	3517	3569	3624	3683	3745	3811	3881	3955	4033	4116	4204	4298	4397	4501	4612	4730	4854	4986	nh	0	156	322	498	685	883	1092	1314	1549
2793	3316	3347	3380	3415	3452	3491	3532	3576	3623	3673	3725	3780	3839	3901	3967	4037	4111	4189	4272	4360	4454	4553	4657	4768	4886	nh	nh	nh	nh	0	166	342	529	727	936	1158	1393
2959	3482	3513	3546	3581	3618	3657	3698	3742	3789	3839	3891	3946	4005	4067	4133	4203	4277	4355	4438	4526	4620	4719	4823	4934	nh	nh	nh	nh	nh	nh	0	176	363	561	770	992	1227
3135	3658	3689	3722	3757	3794	3833	3874	3918	3965	4015	4067	4122	4181	4243	4309	4379	4453	4531	4614	4702	4796	4895	4999	nh	nh	nh	nh	nh	nh	nh	nh	0	187	385	594	816	1051
3322	3845	3876	3909	3944	3981	4020	4061	4105	4152	4202	4254	4309	4368	4430	4496	4566	4640	4718	4801	4889	4983	nh	nh	nh	nh	nh	nh	nh	nh	nh	nh	nh	0	198	407	629	864
3520	4043	4074	4107	4142	4179	4218	4259	4303	4350	4400	4452	4507	4566	4628	4694	4764	4838	4916	4999	nh	nh	nh	nh	nh	nh	nh	nh	nh	nh	nh	nh	nh	nh	0	209	431	666
3729	4252	4283	4316	4351	4388	4427	4468	4512	4559	4609	4661	4716	4775	4837	4903	4973	nh	nh	nh	nh	nh	nh	nh	nh	nh	nh	nh	nh	nh	nh	nh	nh	nh	nh	0	222	457
3951	4474	4505	4538	4573	4610	4649	4690	4734	4781	4831	4883	4938	4997	nh	nh	nh	nh	nh	nh	nh	nh	nh	nh	nh	nh	nh	nh	nh	nh	nh	nh	nh	nh	nh	nh	0	235
4186	4709	4740	4773	4808	4845	4884	4925	4969	nh	nh	nh	nh	nh	nh	nh	nh	nh	nh	nh	nh	nh	nh	nh	nh	nh	nh	nh	nh	nh	nh	nh	nh	nh	nh	nh	nh	0

Frequencies marked as nh in the tables are inaudible either because of auditory limitations, or of the filters as designed into our circuitry.

For the calculation of the required tube lengths, we started from the formula given in Harry Olson, 'Music, Physics and engineering' p.77:

$$f_1 = \frac{1.133\,\pi}{l^2}\sqrt{\frac{QK^2}{\rho}}$$

$$f_2 = 2.758\ f_1$$
$$f_3 = 5.404\ f_1$$
$$f_4 = 8.933\ f_1$$

f = frequency (Hz)
l = length of the tube (cm)
Q = Young's modulus (dyne/cm2)
K = radius of gyration
ρ = density (g/cm3)

Reformulated to units in the SI system this gives:

$$f_1 = \frac{1.133\,\pi}{l^2}\sqrt{\frac{QK^2}{\rho}}$$

$$f_2 = 2.758\ f_1$$
$$f_3 = 5.404\ f_1$$
$$f_4 = 8.933\ f_1$$

f = frequency (Hz)

l = tube length (m)
Q = modulus of elasticity (Pa)
K = radius of gyration (m)
for cilindrical tube:

$$K = \frac{\sqrt{d_i^2 + d_o^2}}{2}$$

di = internal diameter
do = external diameter

for 30/20 tube:
$K = 0.018028$

ρ = density of the material (kg/m3)
for aluminum: 2702 kg.m3

As we use the same material for all tubes (aluminum, AlMgSi 0.5, with density 2.699 kg / liter) and we also keep the thickness the same (30 mm / 20 mm), we can simplify the formula. The part under the square root becomes a constant 91.1529. The factor 1.133 given by Olson, after our measurements for note 84 (f=1046 Hz) appears to be 1.4308.. The deviation is a lot larger than what can be attributed to the precision of our measurements...

After tuning all 36 tubes and feeding the measurements to a curve-fitting program, we came to the following practical formula to calculate the fundamental frequency in function of the length of the tube:

$$l = \sqrt{\frac{1.63935E6 - (3.9439E1\ f)}{f}}$$

l = length of the tube (m)
f = fundamental frequency (Hz)
for 30/20 aluminum tube,
density 2702 kg/m3
modulus of elasticity: 69GPa

The 'constants' in the textbook formula are clearly a function of pitch. We do not have a sound explanation for this result. Maybe the force exerted on the tube by its weight when suspended, contributes to the deviation? More likely: the propagation speed of sound/vibration itself is a function of frequency? Maybe, Olson's formula -blindly taken over a many textbooks on acoustics- was never empirically checked? We do suspect the latter, as nowhere in the literature we find references to empirical data.

Table with exact lengths of the tubes, point of suspension, midi note and frequency for the tubes:

- absolute error on the length measurements > 310mm : 0.5mm
- absolute error on the length measurements < 310mm: 0.02mm
- error on the frequencies measured : 2 cent.

midi note	frequency (Hz)	length (mm)	Suspension point (nodal)	'constant'
72	523	557.0	124.6	1622602
73	554	541.0	121.0	1621452
74	587	526.0	117.6	1624088
75	622	510.0	114.2	1617822
76	659	496.0	110.8	1621245
77	698	481.0	107.8	1614899
78	739	466.5	104.5	1608228
79	783	453.6	101.7	1611045
80	830	440.0	98.6	1606880
81	880	426.5	95.6	1600739
82	932	414.5	92.9	1601271
83	987	402.0	90.1	1595031
84	1046	391.5	87.7	1603227
85	1108	380.0	85.2	1599952
86	1174	369.0	82.7	1598530
87	1244	357.0	80.0	1585465
88	1318	347.5	77.9	1591567
89	1396	336.0	75.3	1576028
90	1479	326.5	73.2	1576647
91	1576	316.8	71.0	1572676
92	1661	307.73	69.0	1572929
93	1760	298.72	67.0	1570512
94	1864	290.36	65.1	1571518
95	1975	281.18	63.0	1561478
96	2093	272.50	61	1551902
97	2217	264.10	59.2	1546331
98	2349	256.62	57.5	1546906

99	2489	249.14	55.7	1544940
100	2637	238.90	53.6	1505020
101	2793	234.22	52.4	1532212
102	2959	227.28	50.8	1528506
103	3135	219.96	49.3	1516788
104	3322	213.20	47.8	1509989
105	3520	206.70	46.3	1503916
106	3729	199.62	44.8	1485937
107	3951	194.08	43.4	1488225
108	4186	188.34	42.0	1484856

The output of the Plus-Minus installation comes from three large megaphone speakers (exponential horns) mounted on the vertical poles. We used such loudspeakers for they have an intrinsic high-pass filter characteristic (thus avoiding feedback by mechanical coupling) at the one hand, and for their pronounced directional radiation pattern. Thus players can clearly hear what they are doing, the sound source being very close to the origin of the vibrations.

The installation can be played by three persons simultaneously. Vibraphone mallets go with the instrument as we found them to sound best. Of course other beaters and materials can be used as well. A side effect, as we discovered soon enough, consists in the possibility to sing in close proximity to the tubes (and the microphones hidden behind them). The vocal sounds than get ring modulated with the tubular bell sounds leading to quite intriguing musical results.

The whole installation is easily transportable and can be set up in less than an hour. It is available for rent from the Logos Foundation.

dr. Godfried-Willem Raes

Technical data:

- Power: 230V ac / 3 A max
- Sound pressure level: <= 90 dBA
- Weight: ca. 50kg
- Sizes: 1400 mm (triangular), height: 1800 mm
- required surface for set up: 16 m2 (4m x 4m):
- Insurance value: 25.000 Euro

Parts list and component data:

- Microphones: ABM-713-RC , Pro Signal. Farnell order code: 206-6499. Datasheet.
- AD633ANZ, Analog Devices multiplier. Farnell order code: 143-38410. . Datasheet
- LF356N op amp. Farnell order nr.: 948-7000. Datasheet
- OP27GPZ, Analog Devices of Burr Brown. Farnell order code: 960-4626. Datasheet
- Texas Instruments, class D amplifier module. Farnell order code: 280-2975. Datasheet
- 24V SMPS power supply unit. Farnell order code: 263-0552. . Datasheet
- BUF634P. Datasheet. (Burr-Brown)

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